Graphs of Functions

The book defines the graph of a function as the set of ordered pairs

$$\{(x, f(x)) \mid x \in A\}$$

where A is the domain of the function.

While accurate, this is a somewhat tortured way to describe a graph. Usually by a graph we mean a visual representation showing those points.
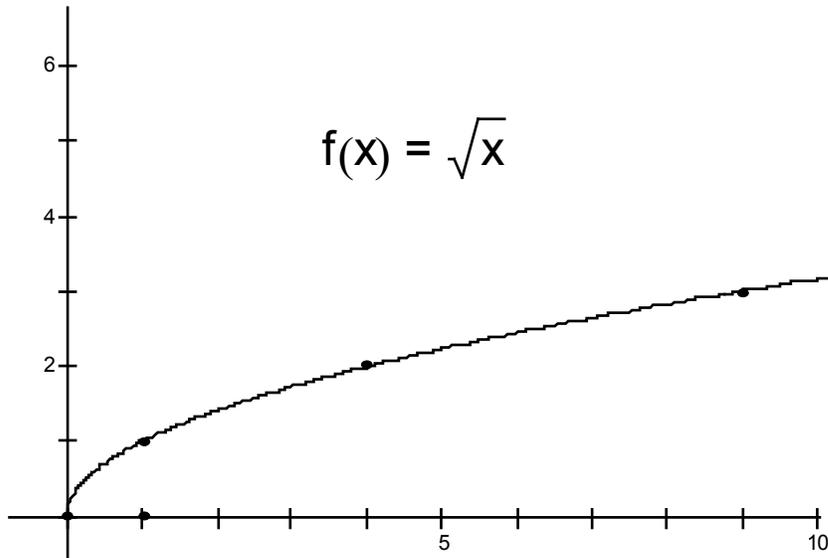
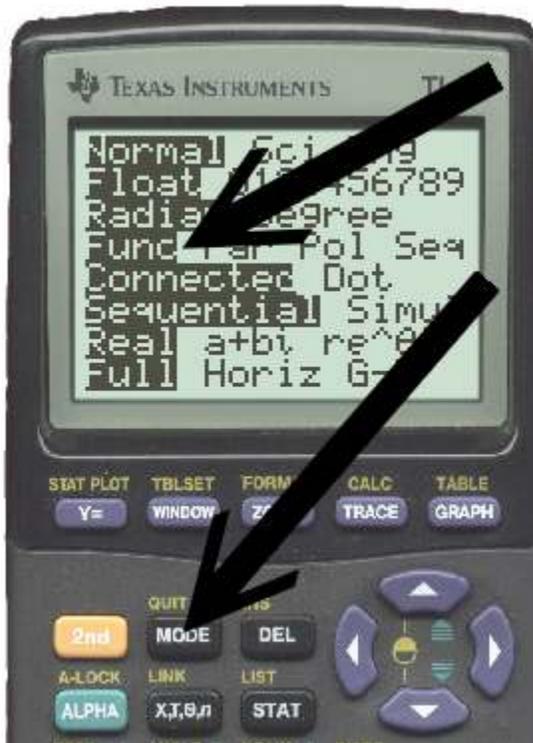There are a number of ways to create a graph given a function

Using paper and pencil

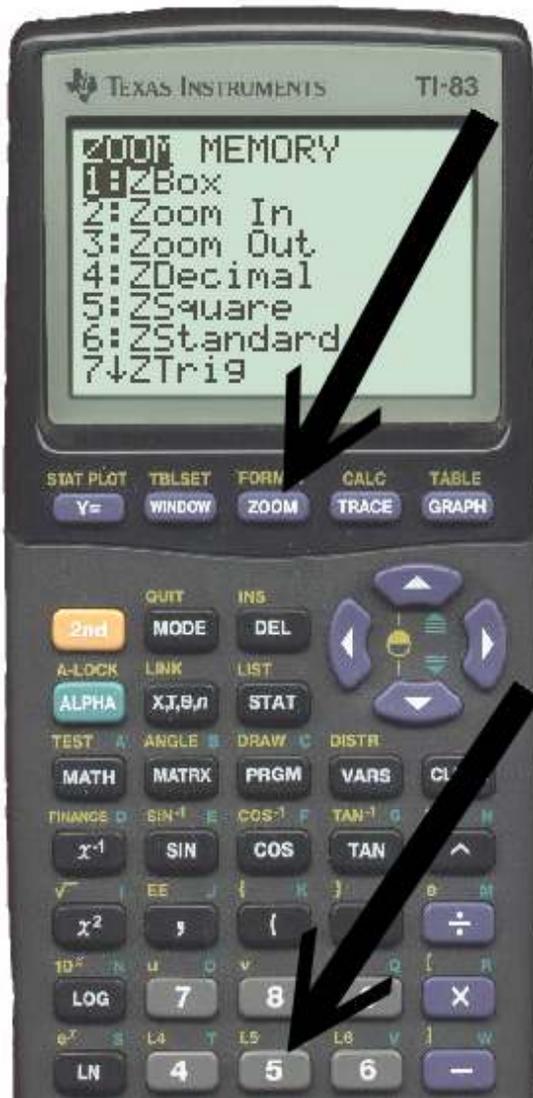Plot points and then draw a line through the graph. (demonstrate)
$$f(x) = \sqrt{x}$$

x	$\sqrt{x}$
0	0
1	1
4	2
9	3

Using a TI-83 graphics calculator or similar model

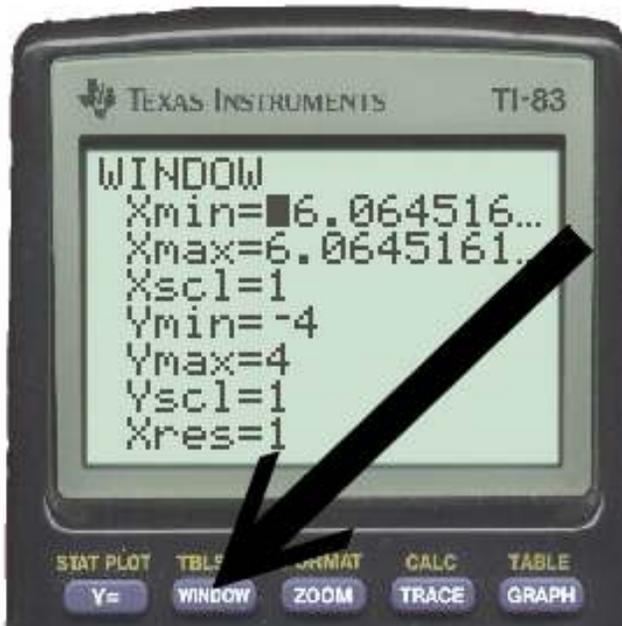Press the [MODE] button and make sure the calculator is in "Func" mode.

Press the [ZOOM] key and then [5] for ZSquare.

This sets up the window axes to the same scale.

Other options in this window allow different configurations, or zooming in and out.

You should experiment with the different options.

You can also press the [WINDOW] key and set the Axis parameters directly.

Xmin is the lowest X value
Xmax is the highest X value
Xscl (X-Scale) controls where the tick marks on the axis go.
    A 1 means at every integer
    A 5 means every 5
Xres determines the X resolution. A larger number will show a smoother graph, but could take more time to display.

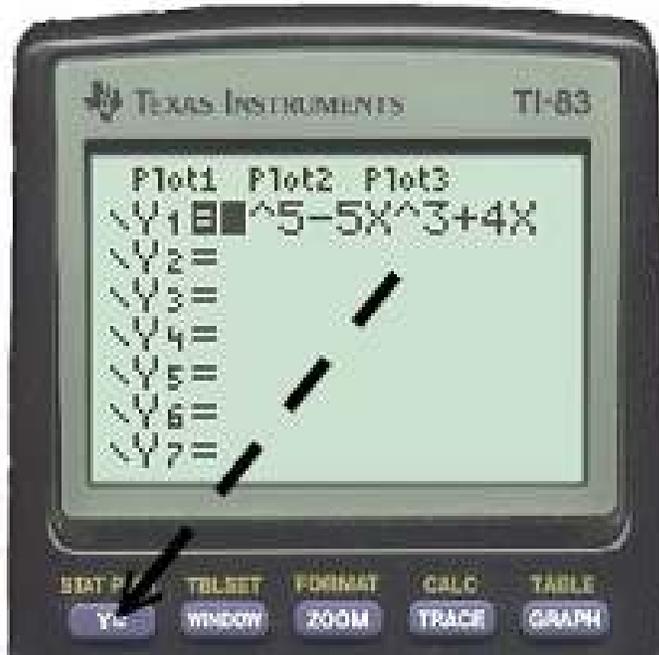The Y parameters work the same way, but with the Y axis.

Be careful when entering a negative number to use the
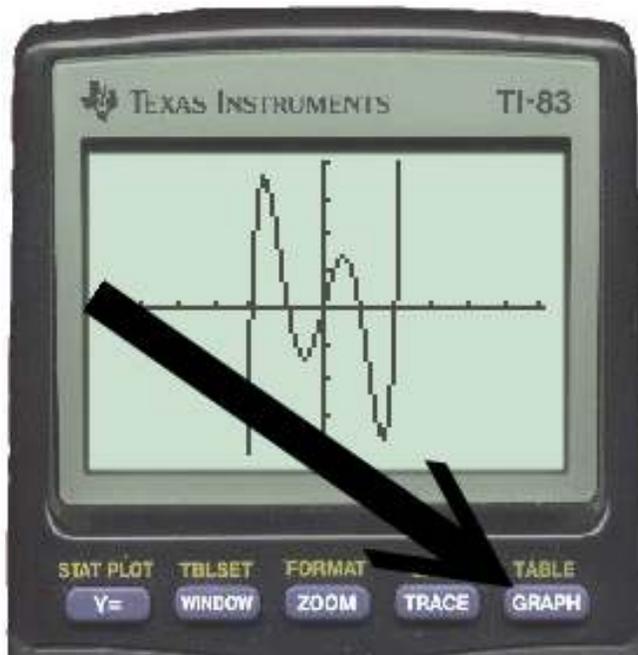
[(-)] Minus button

and not the

[-] subtract button.

Press the [Y=] key to bring up the function editor.

Enter a function into the Y$_1$ Slot using the [X,T,θ,n] key for the independent variable.

You can enter any of the functions accessible at the keyboard, such as $\sqrt{X}$ or using the [

[MATH] and [CATALOG] buttons.

Press the [GRAPH] key to see your Graph.

Note: If the calculator seems stuck, it may just be slow, so give it some time.

If you run out of patience, press [2nd][Off] to turn the calculator off, and then turn it on again.

There are many useful functions you can use once you have your function in the calculator. Examples are:
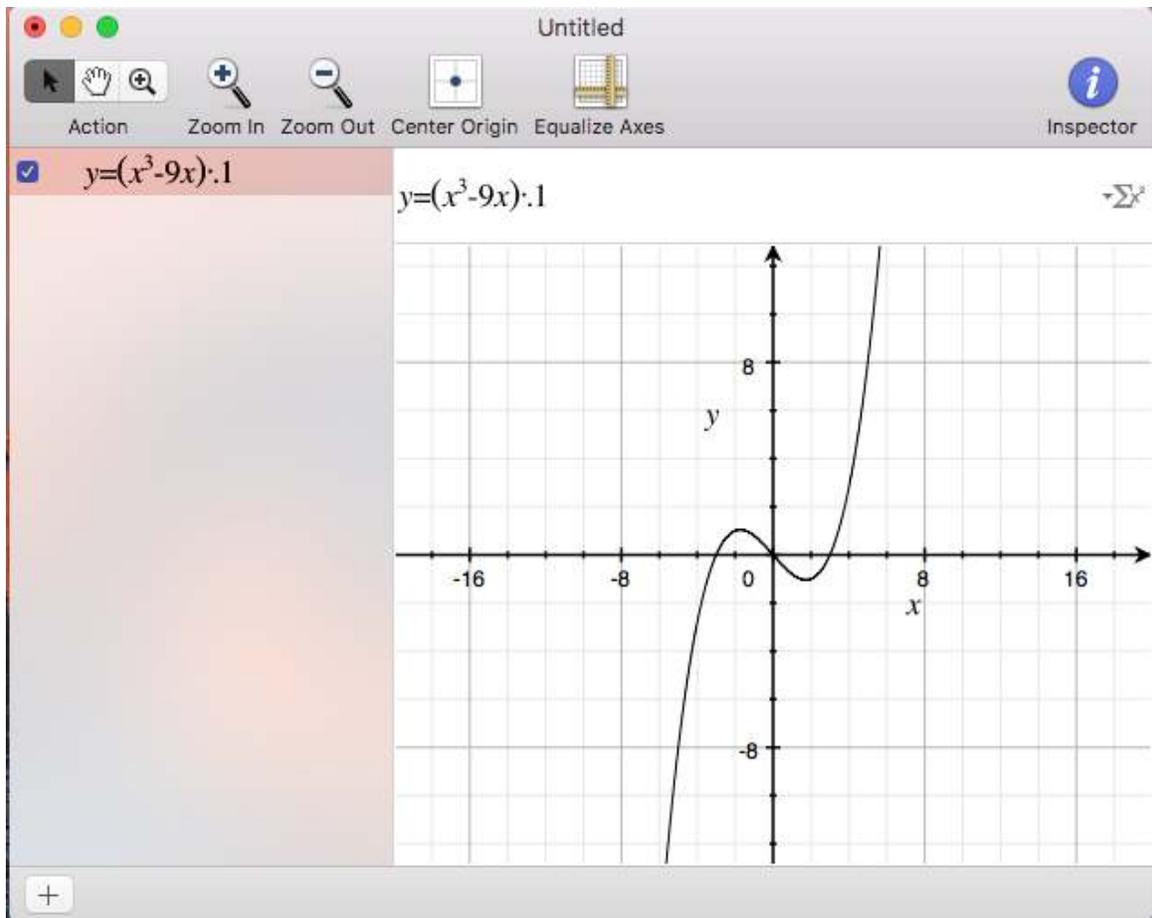
1) Calculate the value of the function at a point

2) Display a table of values

3) Find a zero of the function

4) Find a local minimum or maximum of the function

5) Find the intersection of a function

Do Calculus calculations such as differentiation and integration.
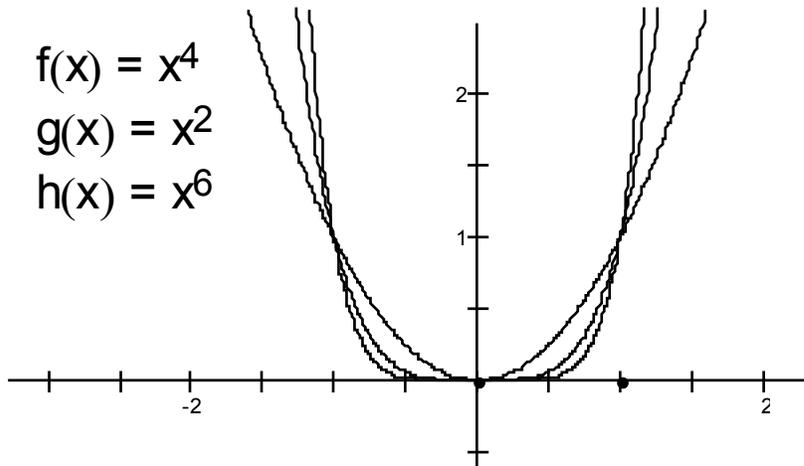
A Computer program eg. Mac grapher
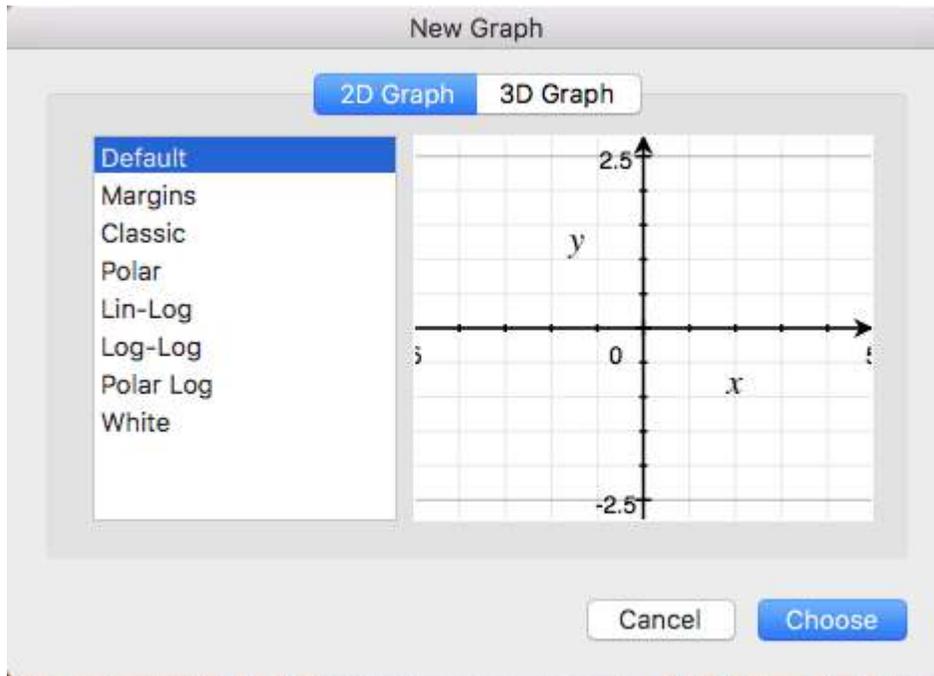
Choose a Graph Type, eg. 2D Graph: Default

Type in a function

Graphing $f(x) = x^n$

Notice that when n is even, the graph is symmetric on the *Y*-axis and always positive in the first and second quadrants.
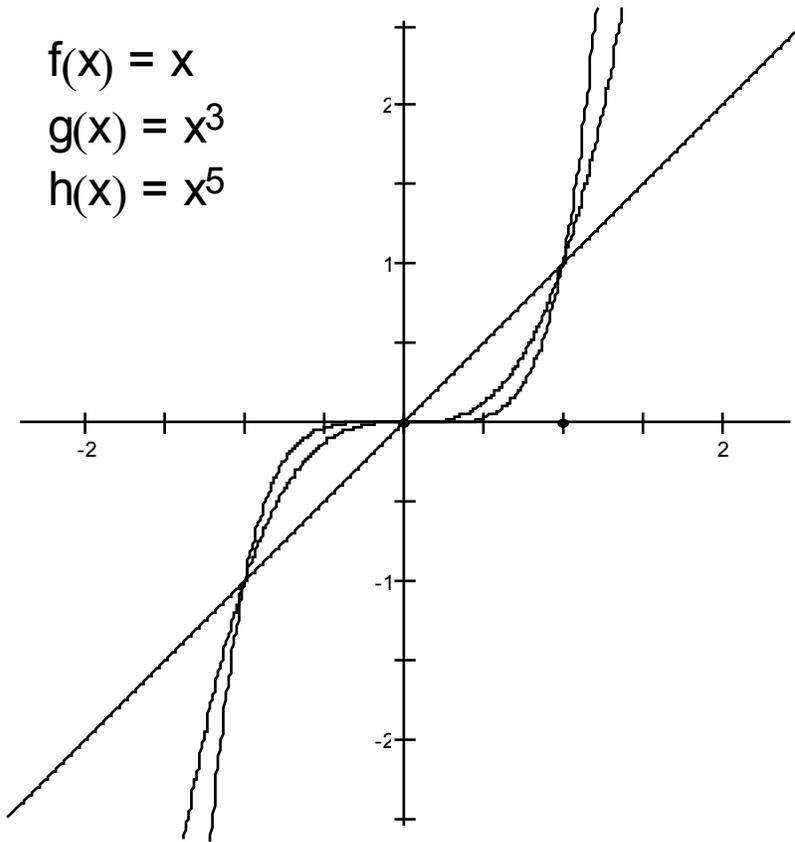
$$f(x) = x^4$$

$$g(x) = x^2$$

$$h(x) = x^6$$

If *n* is odd, the graph is symmetric with the origin (0,0) and it's points are in the first and third quadrants.
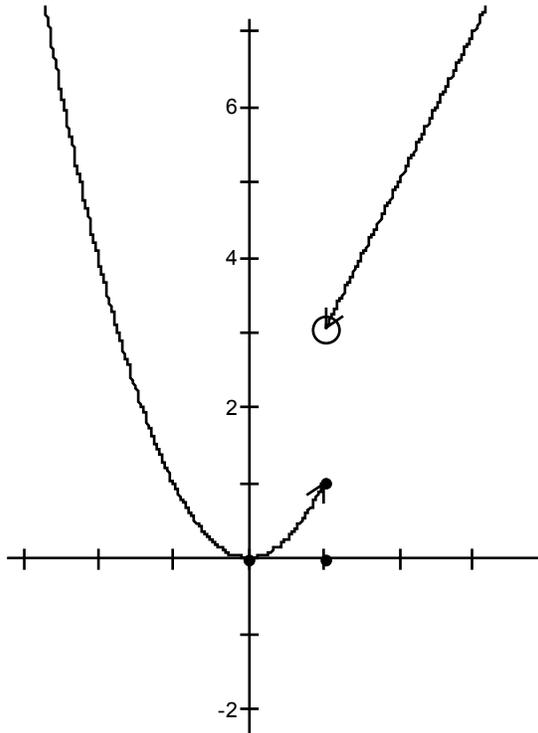
$$f(x) = x$$

$$g(x) = x^3$$

$$h(x) = x^5$$

Graphing a **Piecewise function**

Example:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

Graphing a Step Function

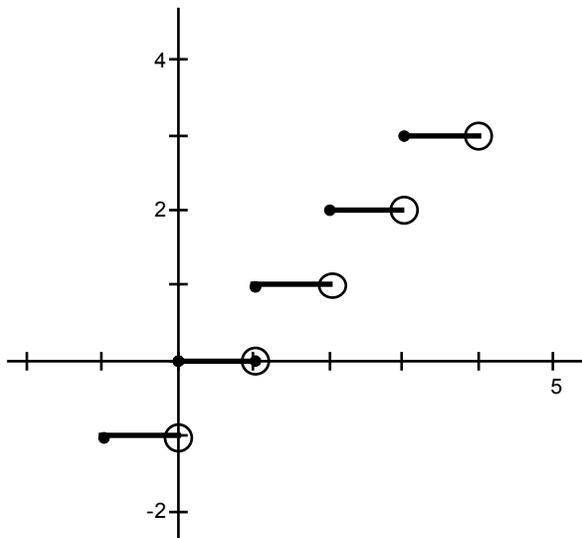The symbol $[\![x]\!]$ means the greatest integer less than or equal to x.

So for example:

$[\![1]\!]=1$
$[\![.5]\!]=0$
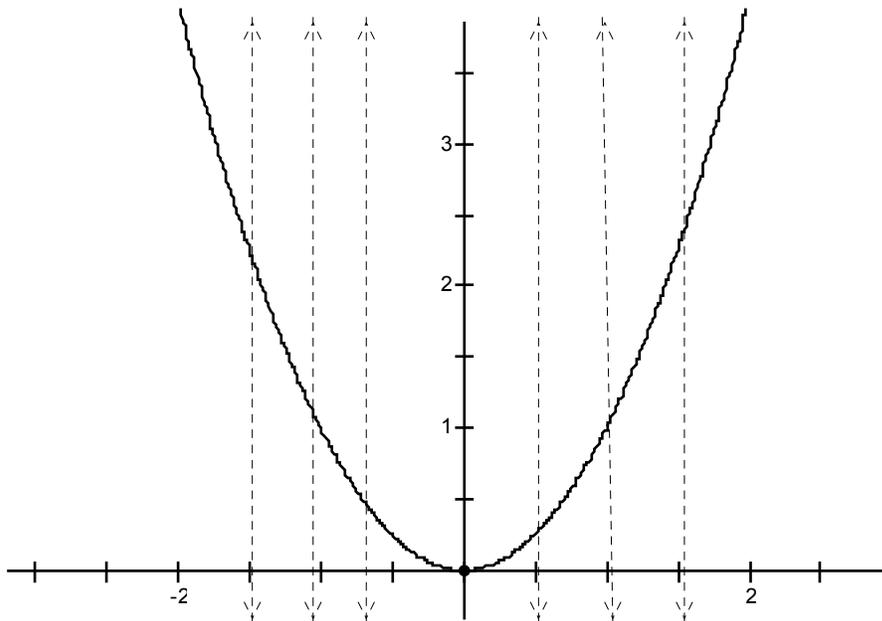$[\![-.5]\!]=-1$

The graph of this function is as follows:

The Vertical Line Test

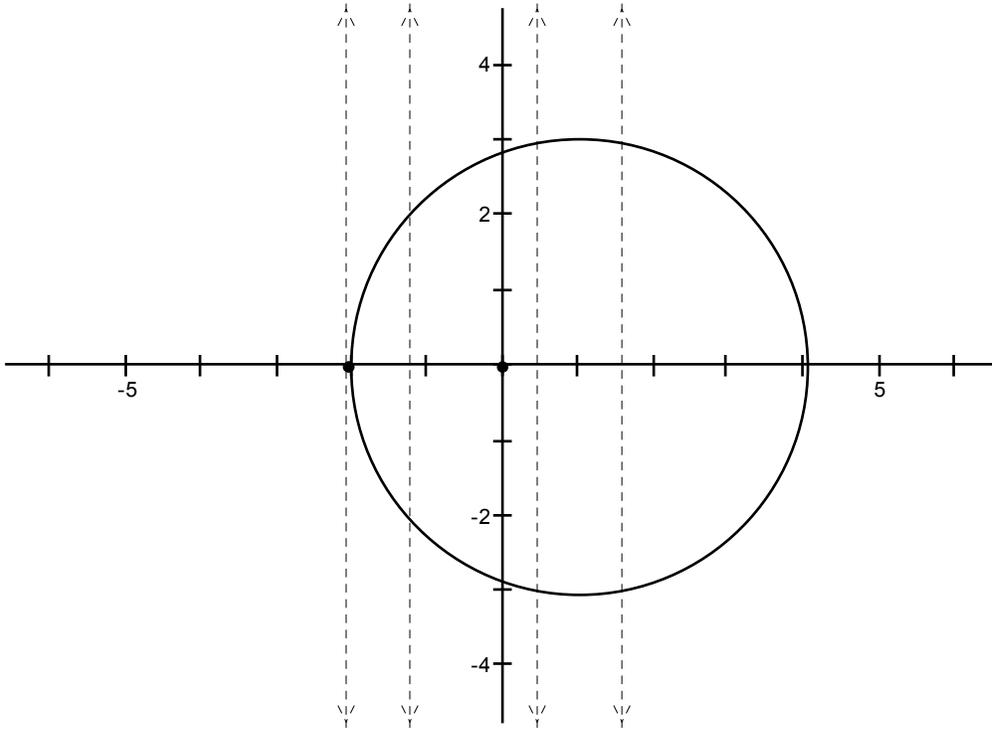Sometimes we want to know whether a graph represents a function.

Recall that a function has one unique value for each *x*.

So if we draw a **vertical line** on the graph anywhere and it intersects the graph in more than one point, it is NOT a function.

Examples:

This graph of a parabola is clearly a function since none of the vertical lines will pass through more than one point.

This graph of a circle clearly fails the vertical line test and is not a function.